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applicable to Q, (3) cosines applicable to P, (4) cosines applicable to Q. But in practice it would be doubtless impossible to separate these terms satisfactorily.

Before leaving the point perhaps I may state my position in other words thus :

Mr. Cowell writes (vol. lxiv. p. 96) : "The hypothesis that the error of semidiameter is a constant is the best that I can make."

I venture to doubt this, even in the circumstances considered by Mr. Cowell. I think he should have considered the expression

$$b_0 + b_1 \cos D + b_2 \cos 2D + \&c.$$

instead of the first term only.

The remainder of Mr. Cowell's paper is an admirable exposition of the principles of his analysis, which I am glad to have drawn from him. If I refrain from comment it is because I am afraid of prolonging the discussion unduly. The two points mentioned seem to me important, and I hope I have made my position with regard to them clear. Mr. Cowell and I have, as already mentioned, discussed these matters without being able to arrive at an agreement, so that he will doubtless represent his views clearly also, and there I hope the matter may be left.

Reply to Professor Turner's further Note. By P. H. Cowell.

1. Let me put

N = instrumental error of transit circle (1851–61)
O = ,, ,, "old transit circle" (1847–50)
L = ,, ,, altazimuth (1851–61)
M = ,, ,, ,, (1847–50)

Then Professor Turner has shown, by processes that I have no objection to, that $N-O-L+M$ varies from 0″·0 on day 17 to +1″·3 on day 21, and then decreases to +0″·8 on day 24, the zero being an arbitrary one (vol. lxiv. p. 411).

Regarding this table merely as proving that the above combination of instrumental error varies with the day of the lunation, it was clearly unnecessary for me to call attention to the absence of days 15 and 16. The importance of these days is, however, considerable when we attempt to consider the law connecting the above observed quantity with the age of the Moon.

I do not wish my chief objections to Professor Turner's first paper to be obscured by a discussion of minor points. What I object to is this. He says $L-M$ may be put equal to zero, and

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he puts N—O instead of N—O—L+M at the top of the table referred to. From some analyses of my own, given on vol. lxiv. p. 580, it may be inferred that the values of N—O are really about one-third of those given by Professor Turner. The other two-thirds must apparently be ascribed to L—M, and give some indication of the accidental errors to which the altazimuth was liable.

2. The formula to which I equate the errors is

$$\begin{aligned}\epsilon \pm \mu + \delta_1 \sin D + \delta_2 \sin 2D + \ldots \\ + \Delta_1 \cos D + \Delta_2 \cos 2D + \ldots\end{aligned}$$

The formula used by Airy (*Memoirs* R.A.S., vol. xxix. p. 4) is

$$\delta_1 \sin D + \delta_2 \sin 2D$$

Professor Turner suggests

$$\begin{aligned}\pm(\mu + b_1 \cos D + b_2 \cos 2D + \ldots \\ + a_1 \sin D + a_2 \sin 2D + \ldots) \\ + \epsilon + \delta_1 \sin D + \delta_2 \sin 2D + \ldots \\ + \quad \Delta_1 \cos D + \Delta_2 \cos 2D + \ldots\end{aligned}$$

My formula is therefore intermediate between that used by Airy and that suggested by Professor Turner.

There is a phrase, "necessary and sufficient," in constant use by mathematicians. I maintain that Airy's formula is insufficient and Professor Turner's unnecessary. Airy's is insufficient because the deduced value of the parallactic inequality is clearly erroneous if his semidiameter is erroneous. Airy has, in effect, therefore made his parallactic inequality depend, not upon the 10,000 observations that he reduced, but upon the 100 observations that he used for semidiameter.

Each one of the additional terms proposed by Professor Turner can, on the other hand, be shown to be unnecessary. I will take one only as an example, $\pm b_1 \cos D$. This may be replaced by

$$b_1(\pm 1{\cdot}000 - 1{\cdot}056 \sin D + 0{\cdot}478 \sin 2D)$$

for the expression that I have just written down differs from $\pm b_1 \cos D$ by considerably less than $\frac{1}{10} b_1$ from $D = 70°$ to $D = 290°$.

A similar remark being true of all Professor Turner's other terms as well, it is clear that he adds nothing to the generality of my hypothesis by assuming that the semidiameter is

$$\begin{aligned}\mu + b_1 \cos D + b_2 \cos 2D \\ + a_1 \sin D + a_2 \sin 2D\end{aligned}$$

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rather than by assuming that it is

$$\mu \pm (\alpha_1 \sin D + \alpha_2 \sin 2D \\ + \beta_1 \cos D + \beta_2 \cos 2D)$$

in which case the inequalities of semidiameter merge into the inequalities of longitude.

My point can also be demonstrated thus. Remove $\pm\mu$ from my formula, and it becomes the most general expression possible for errors periodic in the lunation and free from discontinuity. The proper provision to make for a single discontinuity is to introduce a single additional unknown quantity.

The Coefficient of the Principal Term in the Moon's Latitude.

By P. H. Cowell.

1. The coefficient of sin F in Hansen's tabular latitude transformed to Delaunay's form is	18461″·65
2. The mean value of the observed *minus* tabular errors of 5646 observations from 1847 to 1901, each multiplied by 2 sin F, is... ...	−0″·27±0·03
3. The correction to the deduced coefficient, on account of the excess +0″·11 sin (F−D) −0″·05 sin (F+D) of Hansen's tables over Brown's coefficients (*Monthly Notices*, vol. lxv. 1905 January), is	−0·03
	18461·35±0·03
4. Estimated correction to reduce to Pulkowa refractions...	0·00±0·02
5. Estimated effect of planetary terms of period nearly equal to that of F	0·00±0·04
Concluded coefficient...	18461·35±0·05

In paragraph 2 the accidental error is estimated from the analogous investigations for the longitude.

In paragraph 4 account is taken of the fact that Stone's refractions were used during the years 1868–1877, in which the longitude of the node decreased from 150° to −50°.

In paragraph 5, as the planetary perturbations in latitude have not yet been computed, an estimate of their possible effect was based upon the fact that the maximum effect of the planetary terms upon the principal elliptic inequality as deduced from the observations of any continuous period of fifty-four years is 0″·10.